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The spontaneous generation of magnetic fields at high temperature in a supersymmetric theory

V.I. Demchik[a], V.V. Skalozub[b]

Dniepropetrovsk National University, 49050 Dniepropetrovsk, Ukraine

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Abstract. The spontaneous generation of magnetic and chromomagnetic fields at high temperature in the minimal supersymmetric standard model is investigated. The consistent effective potential including the one-loop and the daisy diagrams of all bosons and fermions is calculated and the magnetization of the vacuum is observed. The mixing of the generated fields due to the quark and s-quark loop diagrams and the role of superpartners are studied in detail. It is found that the contribution of these diagrams increases the magnetic and chromomagnetic field strengths as compared with the case of a separate generation of fields. The magnetized vacuum state is found to be stable due to the magnetic masses of gauge fields included in the daisy diagrams. Applications of the results obtained are discussed. A comparison with the standard model case is given.

1 Introduction

The possible existence of strong magnetic fields in the early universe is one of the most interesting problems in high-energy physics. Different mechanisms of the fields at work at different stages of the universe's evolution were proposed. These mechanisms as well as the role of strong magnetic fields have been discussed in the surveys of [1–3]. In particular, primordial magnetic fields, being implemented in a cosmic plasma, may serve as the seeds for the present extra-galaxy fields.

The spontaneous vacuum magnetization at high temperature is one of the mechanisms mentioned. It was already investigated both in pure $SU(2)$ gluodynamics [4–6] and in the standard model (SM) [7] where the possibility of this phenomenon has been shown. The stability of the magnetized vacuum state was also studied [6]. The magnetization takes place for the non-abelian gauge fields due to vacuum dynamics. In fact, this is one of the distinguishable features of asymptotically free theories [6,8]. In [4–6] the fermion contributions were not taken into consideration. However, at high temperature they affect the vacuum considerably. Quarks possess both electric and color charges, and therefore the quark loops change the strengths of the simultaneously generated magnetic and chromomagnetic fields [7].

In a supersymmetric theory new peculiarities should be accounted for. First there is the influence of superpartners. These particles having a low spin have to decrease the generated magnetic field strengths. Second, s-quarks also possess electric and color charges, so the interdependence of magnetic and chromomagnetic fields is expected to be stronger as well as the fields due to their vacuum loops. Because of this mixing some specific configurations of the fields must be produced at high temperature.

In the present paper the spontaneous vacuum magnetization is investigated in the minimal supersymmetric standard model (MSSM) of the elementary particles. All boson and fermion fields are taken into consideration. In the MSSM there are two kinds of non-abelian gauge fields – the $SU(2)$ weak isospin gauge fields responsible for weak interactions and the $SU(3)$ gluons mediating the strong interactions. Magnetic and chromomagnetic fields are related to these symmetry groups, respectively. To elaborate this problem we calculate the effective potential (EP) including the one-loop and the daisy diagram contributions in constant abelian chromomagnetic and magnetic fields, $H_{\rm c} = \text{const}$ and $H = \text{const}$, at high temperatures. The values of the generated field strengths are found as the minimum position of the EP in the field strength plane $(H, H_{\rm c})$.

Let us note the advantages of the approximation used. The EP of the background abelian magnetic fields is a gauge fixing independent one, while the daisy diagrams account for the most essential long-range corrections at high temperature. Therefore, such a type of EP includes the leading and the next-to-leading terms in the coupling constants. Moreover, as it was shown in [6,9], the daisy diagrams of the charged gluons and the W-bosons with their magnetic masses taken into consideration make the vacuum with non-zero magnetic fields stable at high temperatures. This stability reflects the consistency of the ap-

[a] e-mail: vadimdi@yahoo.com
[b] e-mail: skalozub@ff.dsu.dp.ua

proximation. The EP of this type was used recently in investigations of the electroweak phase transition in an external hypercharge magnetic field [10] and the spontaneous generation of magnetic and chromomagnetic fields in the SM [7]. The obtained results are in good agreement with the non-perturbative calculations carried out in [11, 12]. This approximation will be used in what follows.

The abelian hypercharge magnetic field is not generated spontaneously. So, in our investigation we shall consider the non-abelian constituent of the magnetic field coming from the $SU(2)$ gauge group. The generation mechanisms of the hypermagnetic field were studied in [9,13]. Below it will be shown that at high temperatures either strong magnetic or chromomagnetic fields are generated in the MSSM, similarly to the SM. The sector additional to the SM sector of the MSSM, the s-particle sector, does not suppress this effect. It just decreases the strengths of the generated fields. These fields are stable in the approximation adopted due to the magnetic masses $m^2_{\mathrm{transversal}} \sim (gH)^{1/2}T$ of the gauge field transversal modes [14]. In this way a consistent picture of the magnetized vacuum state is derived.

The contents of this paper are as follows. In Sect. 2 the contributions of all bosons and fermions to the EP $v'(H,T)$ of external magnetic and chromomagnetic fields are calculated in a form convenient for numeric investigations. In Sect. 3 the field strengths are calculated. A discussion and concluding remarks are given in Sect. 4.

2 Basic formulae

The full Lagrangian of the MSSM can be written as ([15])

$$\mathcal{L} = L_{\mathrm{gs}} + L_{\bar{\mathrm{gs}}} + L_{\mathrm{leptons}} + L_{\mathrm{sleptons}} + L_q + L_{sq} + L_{\mathrm{Higgs}} + L_{\mathrm{higgsino}} + L_{\mathrm{int}} + L_{\mathrm{SSB}} + L_{\mathrm{gf}}. \quad (1)$$

Here, L_{gs} and $L_{\bar{\mathrm{gs}}}$ is the kinetic part of gauge bosons and gauginos, correspondingly; L_{leptons}, L_{sleptons}, L_q and L_{sq} give the kinetic part of the matter (fermions and s-fermions) fields and their interaction Lagrangians; L_{Higgs} is the kinetic part and the Higgs interactions with gauge bosons and gauginos; L_{int} contains the interaction terms; L_{SSB} is the soft-symmetry breaking (SSB) part; L_{gf} is the gauge fixing terms.

In the MSSM there are mixings in the higgsino and gaugino sector resulting in the presence of chargino (mixing of charged higgsino and gaugino) and neutralino (mixing of neutral higgsino and gaugino) in the theory. Since the electrical and color neutral particles are not interacting with magnetic and chromomagnetic fields, we will not take the neutralino into consideration.

The MSSM Lagrangian of the gauge boson sector is [15]

$$L_{\mathrm{gs}} = -\frac{1}{4}F^{\alpha}_{\mu\nu}F^{\mu\nu}_{\alpha} - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} - \frac{1}{4}\mathbf{F}^{\alpha}_{\mu\nu}\mathbf{F}^{\mu\nu}_{\alpha}, \quad (2)$$

where the following standard notation is introduced:

$$\begin{aligned} F^{\alpha}_{\mu\nu} &= \partial_\mu A^\alpha_\nu - \partial_\nu A^\alpha_\mu + g\epsilon^{abc}A^b_\mu A^c_\nu, \quad (3)\\ G_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu,\\ \mathbf{F}^{\alpha}_{\mu\nu} &= \partial_\mu \mathbf{A}^\alpha_\nu - \partial_\nu \mathbf{A}^\alpha_\mu + g_s f^{abc}\mathbf{A}^b_\mu \mathbf{A}^c_\nu. \end{aligned}$$

The fields corresponding to the gauge W-, Z-bosons and photons, respectively, are

$$\begin{aligned} W^{\pm}_\mu &= \frac{1}{\sqrt{2}}(A^1_\mu \pm \mathrm{i}A^2_\mu), \quad (4)\\ Z_\mu &= \frac{1}{\sqrt{g^2+g'^2}}(gA^3_\mu - g'B_\mu),\\ A_\mu &= \frac{1}{\sqrt{g'A^3_\mu + gB_\mu}}, \end{aligned}$$

and $\mathbf{A}^\alpha_\mu$ is the gluon field.

As usually, the introduction of gauge fields is done by replacing all derivatives in the Lagrangian with the covariant ones,

$$\partial_\mu \to \mathcal{D}_\mu = \partial_\mu + \mathrm{i}g\frac{\tau^\alpha}{2}A^\alpha_\mu + \mathrm{i}g_s\frac{\lambda^\alpha}{2}\mathbf{A}^\alpha_\mu. \quad (5)$$

Here τ^α and λ^α stand for the Pauli and the Gell-Mann matrices, respectively.

In the $SU(2)$ sector there is only one magnetic field, the third projection of the gauge field. In the $SU(3)_c$ sector there are two possible chromomagnetic fields connected with the third and the eighth generators of the group.

For simplicity, in what follows we shall consider the field associated with the third generator of $SU(3)_c$.

To introduce the interaction with classical magnetic and chromomagnetic fields, we split the potentials in two parts:

$$\begin{aligned} A_\mu &= \bar{A}_\mu + A^{\mathrm{R}}_\mu, \quad (6)\\ \mathbf{A}_\mu &= \bar{\mathbf{A}}_\mu + \mathbf{A}^{\mathrm{R}}_\mu, \end{aligned}$$

where A^{R} and $\mathbf{A}^{\mathrm{R}}$ describe the radiation fields and $\bar{A} = (0,0,Hx^1,0)$ and $\bar{\mathbf{A}} = (0,0,\mathbf{H}_3x^1,0)$ correspond to the constant magnetic and chromomagnetic fields directed along the third axes in the space and in the internal color and isospin spaces.

To construct the total EP we used the general relativistic renormalizable gauge which is set by the following gauge fixing conditions [16]:

$$\begin{aligned} &\partial_\mu W^{\pm\mu} \pm \mathrm{i}e\bar{A}_\mu W^{\pm\mu} \mp \mathrm{i}\frac{g\phi_{\mathrm{c}}}{2\xi}\phi^{\pm} = C^{\pm}(x), \quad (7)\\ &\partial_\mu Z^\mu - \frac{\mathrm{i}}{\xi'}(g^2+g'^2)^{1/2}\phi_{\mathrm{c}}\phi_Z = C^Z(x),\\ &\partial_\mu \mathbf{A}^\mu + \mathrm{i}g_s\bar{\mathbf{A}} = \mathbf{C}(x), \end{aligned}$$

where $e = g\sin\theta_{\mathrm{W}}$, $\tan\theta_{\mathrm{W}} = g'/g$, $\phi^{\pm}$ and ϕ_Z are the Goldstone fields, ξ and ξ' are the gauge fixing parameters, $C^{\pm}$ and C^Z are arbitrary functions and ϕ_{c} is the value of the scalar field condensate. Setting $\xi, \xi' = 0$ we choose the unitary gauge. In the restored phase the scalar field condensate $\phi_{\mathrm{c}} = 0$ and the equations (7) are simplified.

The values of the macroscopic magnetic and chromomagnetic fields generated at high temperature will be calculated by minimization of the thermodynamic potential Ω which is introduced as follows:

$$\Omega = -\frac{1}{\beta}\log Z, \quad (8)$$

$$Z = \mathrm{Tr}\exp(-\beta\mathcal{H}), \quad (9)$$

where Z is the partition function, and $\mathcal{H}$ is the Hamiltonian of the system. The trace is calculated over all physical states.

To obtain the EP one has to rewrite (8) as a sum in quantum states calculated near the non-trivial classical solutions A^{ext} and $\mathbf{A}^{\mathrm{ext}}$. This procedure is well-described in the literature (see, for instance, [6,17,18]), and the result can be written in the form

$$V = V^{(1)}(H, \mathbf{H}_3, T) + V^{(2)}(H, \mathbf{H}_3, T) + \dots + V_{\mathrm{daisy}}(H, \mathbf{H}_3, T) + \dots, \quad (10)$$

where $V^{(1)}$ is the one-loop EP; the other terms present the contributions of two-, three-, etc. loop corrections.

Among these terms there are some responsible for the dominant contributions of long distances at high temperature – the so-called daisy or ring diagrams (see, for example, [17]). This part of the EP, $V_{\mathrm{daisy}}(H, \mathbf{H}_3, T)$, is nonzero in the case that massless states appear in a system. The ring diagrams have to be calculated when the vacuum magnetization at finite temperature is investigated. In fact, one first must assume that the fields are nonzero, calculate the EP $V(H, \mathbf{H}_3, T)$ and after that check whether its minimum is located at non-zero H and $\mathbf{H}_3$. On the other hand, if one investigates problems in the applied external fields, the charged fields become massive with the masses depending on $\sim (gH)^{1/2}$, $\sim (g_s\mathbf{H}_3)^{1/2}$ and have to be omitted.

The one-loop contribution to the EP is given by the expression

$$V^{(1)} = -\frac{1}{2}\mathrm{Trlog}G^{ab}, \quad (11)$$

where G^{ab} stands for the propagators of all quantum fields $W^{\pm}$, $\mathbf{A}$, ... in the background fields H and $\mathbf{H}_3$. In the proper time formalism, the s-representation, the calculation of the trace can be carried out in accordance with the formula [19]

$$\mathrm{Trlog}G^{ab} = -\int_0^\infty \frac{\mathrm{d}s}{s}\mathrm{tr}\exp(-\mathrm{i}sG^{-1}_{ab}). \quad (12)$$

Details of calculations based on the s-representation and formula (13) can be found in [20–22].

To incorporate the temperature into this formalism in a natural way, we make use of the method of [20] which connects the Green functions at zero temperature with the Matsubara Green functions,

$$G_k^{ab}(x, x'; T) = \sum_{-\infty}^{+\infty}(-1)^{(n+[x])\sigma_k}G_k^{ab}(x - [x]\beta u, x' - n\beta u), \quad (13)$$

where G_k^{ab} is the corresponding function at $T = 0$, $\beta = 1/T$, $u = (0, 0, 0, 1)$, $[x]$ denotes the integer part of x_4/β, $\sigma_k = 1$ in the case of physical fermions and $\sigma_k = 0$ for boson and ghost fields. The Green functions on the right-hand side of (13) are the matrix elements of the operators G_k computed in the states $|x', a\rangle$ at $T = 0$, and on the left-hand side the operators are averaged over the states with $T \neq 0$. The corresponding functional spaces U^0 and U^{T} are different but in the limit of $T \to 0$ U^{T} transforms into U^0.

The terms with $n = 0$ in (13) and (11) give the zero-temperature expressions for the Green functions and the EP V', respectively. So we can split the latter into two parts:

$$V'(H, \mathbf{H}_3, T) = V'(H, \mathbf{H}_3) + V'_\tau(H, \mathbf{H}_3, T). \quad (14)$$

The standard procedure to account for the daisy diagrams is to substitute the tree level Matsubara Green functions in (11), $[G_i^{(0)}]^{-1}$, by the full propagator $G_i^{-1} = [G_i^{(0)}]^{-1} + \Pi(H, T)$ (see for details [6,17,18]), where the last term is the polarization operator at finite temperature in the field taken at zero longitudinal momentum $k_l = 0$.

Omitting the detailed calculations we notice that the exact one-loop EP is transformed into the EP which contains the daisy diagrams as well as the one-loop diagrams if one adds to the exponent a term containing the temperature dependent mass of a particle.

It is convenient for what follows to introduce the dimensionless quantities: $x = H/H_0$ ($H_0 = M_W^2/e$), $y = \mathbf{H}_3/\mathbf{H}_3^0$ ($\mathbf{H}_3^0 = M_W^2/g_s$), $\beta = M_W/T$ and $v = V/H_0^2$.

The total EP consists of several terms:

$$v' = \frac{x^2}{2} + \frac{y^2}{2} + v'_{\mathrm{leptons}} + v'_q + v'_{W-\mathrm{bosons}} + v'_{\mathrm{gluons}} + v'_{\mathrm{sleptons}} + v'_{sq} + v'_{\mathrm{charginos}} + v'_{\mathrm{gluinos}}. \quad (15)$$

These terms can be written down as follows (in dimensionless variables).

SM sector

Leptons

$$v'_{\mathrm{leptons}} = -\frac{1}{4\pi^2}\sum_{n=1}^{\infty}(-1)^n\int_0^\infty \frac{\mathrm{d}s}{s^3} \times \mathrm{e}^{-(m^2_{\mathrm{leptons}}s + (\beta^2n^2)/(4s))}(xs\mathrm{Coth}(xs) - 1). \quad (16)$$

Quarks

$$v'_q = -\frac{1}{4\pi^2}\sum_{f=1}^{6}\sum_{n=1}^{\infty}(-1)^n\int_0^\infty \frac{\mathrm{d}s}{s^3}\mathrm{e}^{-(m_f^2s + (\beta^2n^2)/(4s))} \times (q_fxs\mathrm{Coth}(q_fxs)\cdot ys\mathrm{Coth}(ys) - 1). \quad (17)$$

W-bosons

See [23].

$$v'_W = -\frac{x}{8\pi^2}\sum_{n=1}^{\infty}\int_0^\infty \frac{\mathrm{d}s}{s^2}\mathrm{e}^{-(m_W^2s + (\beta^2n^2)/(4s))} \times \left[\frac{3}{\mathrm{Sinh}(xs)} + 4\mathrm{Sinh}(xs)\right]. \quad (18)$$

Gluons

See [6].

$$v'_{\mathrm{gluons}} = -\frac{y}{4\pi^2}\sum_{n=1}^{\infty}\int_0^{\infty}\frac{\mathrm{d}s}{s^2}\mathrm{e}^{-(m^2_{\mathrm{gluons}}s+(\beta^2n^2)/(4s))} \times \left[\frac{1}{\mathrm{Sinh}(ys)}+2\mathrm{Sinh}(ys)\right]. \quad (19)$$

MSSM sector

s-leptons

$$v'_{\mathrm{sleptons}} = -\frac{3}{4\pi^2}\sum_{n=1}^{\infty}\int_0^{\infty}\frac{\mathrm{d}s}{s^3}\mathrm{e}^{-(m^2_{\mathrm{sleptons}}s+(\beta^2n^2)/(4s))} \times \left[\frac{xs}{\mathrm{Sinh}(xs)}-1\right]. \quad (20)$$

s-quarks

$$v'_{sq} = -\frac{1}{8\pi^2}\sum_{n=1}^{\infty}\int_0^{\infty}\frac{\mathrm{d}s}{s^3}\mathrm{e}^{-(m^2_{sq}s+(\beta^2n^2)/(4s))} \times \left[\frac{q_f xs\cdot ys}{\mathrm{Sinh}(q_f xs)\cdot\mathrm{Sinh}(ys)}-1\right]. \quad (21)$$

Charginos

$$v'_{\mathrm{charginos}} = -\frac{1}{4\pi^2}\sum_{n=1}^{\infty}(-1)^n\int_0^{\infty}\frac{\mathrm{d}s}{s^3} \times \mathrm{e}^{-(m^2_{\mathrm{charginos}}s+(\beta^2n^2)/(4s))}(xs\mathrm{Coth}(xs)-1). \quad (22)$$

Gluinos

$$v'_{\mathrm{gluinos}} = -\frac{1}{4\pi^2}\sum_{n=1}^{\infty}(-1)^n\int_0^{\infty}\frac{\mathrm{d}s}{s^3} \times \mathrm{e}^{-(m^2_{\mathrm{gluinos}}s+(\beta^2n^2)/(4s))}(ys\mathrm{Coth}(ys)-1). \quad (23)$$

Here, m_{leptons}, m_f, m_W, m_{gluons}, m_{sleptons}, m_{sq}, $m_{\mathrm{charginos}}$ and m_{gluinos} are the dimensionless temperature masses of leptons, quarks, W-bosons, gluons, s-leptons, s-quarks, charginos and gluinos, respectively;

$$q_f = \left(\frac{2}{3},-\frac{1}{3},-\frac{1}{3},\frac{2}{3},-\frac{1}{3},\frac{2}{3}\right)$$

are the charges of the quarks.

Since we investigate the high-temperature effects connected with the presence of external fields, we used terms leading in temperature of the Debye masses of the particles only [6,23].

In the present analysis the temperature masses of leptons, quarks, s-leptons, s-quarks, charginos, gluinos are taken as follows [7]:

$$m^2_{\mathrm{leptons}} = \left(\frac{e}{\beta}\right)^2, \quad m^2_f = \left(\frac{e}{\beta}\right)^2, \quad (24)$$
$$m^2_{\mathrm{sleptons}} = \left(\frac{e}{\beta}+\frac{M_{sl}}{M_W}\right)^2, \quad m^2_{sq} = \left(\frac{e}{\beta}+\frac{M_{sq}}{M_W}\right)^2,$$
$$m^2_{\mathrm{charginos}} = \left(\frac{e}{\beta}+\frac{M_{ch}}{M_W}\right)^2, \quad m^2_{\mathrm{gluinos}} = \left(\frac{e}{\beta}+\frac{M_{gl}}{M_W}\right)^2,$$

where the masses from the SSB terms are taken as the low experimental limits of the corresponding particle masses [24]

$$M_{sl} = 40\,\mathrm{GeV}, \quad M_{sq} = 176\,\mathrm{GeV}, \quad M_{ch} = 62\,\mathrm{GeV}, \quad M_{gl} = 154\,\mathrm{GeV}. \quad (25)$$

As it was established in numeric computation the spontaneous generation of fields depends on the SSB masses fairly weakly. Even in the case of zero SSB masses there is the generation of magnetic and chromomagnetic fields. In the limit of infinite SSB masses the picture conforms to the SM case.

The temperature masses of gluons and W-bosons are [6,7,14]

$$m^2_W = 15\alpha_{\mathrm{EW}}\frac{x^{1/2}}{\beta}, \quad m^2_{\mathrm{gluons}} = 15\alpha_{\mathrm{s}}\frac{y^{1/2}}{\beta}; \quad (26)$$

α_{EW} and α_{s} are the electroweak and the strong interaction couplings, respectively.

In one-loop order, the neutral gluon contribution is a trivial $\mathbf{H}_3$-independent constant which can be omitted. However, these fields are long-range states and they do give a $\mathbf{H}_3$-dependent EP through the correlation corrections depending on the temperature and field. We include the longitudinal neutral modes only because their Debye masses $\Pi^0(y,\beta)$ are non-zero. The corresponding EP is [6]

$$v_{\mathrm{ring}} = \frac{1}{24\beta^2}\Pi^0(y,\beta) - \frac{1}{12\pi\beta}\left(\Pi^0(y,\beta)\right)^{3/2} + \frac{\left(\Pi^0(y,\beta)\right)^2}{32\pi^2}\left[\log\left(\frac{4\pi}{\beta(\Pi^0(y,\beta))^{1/2}}\right)+\frac{3}{4}-\gamma\right]; \quad (27)$$

γ is Euler's constant, $\Pi^0(y,\beta) = \Pi^0_{00}(k=0,y,\beta)$ is the zero–zero component of the neutral gluon field polarization operator calculated in the external field at finite temperature and taken at zero momentum [6],

$$\Pi^0(y,\beta) = \frac{2g^2}{3\beta^2} - \frac{y^{1/2}}{\pi\beta} - \frac{y}{4\pi^2}. \quad (28)$$

Equations (15)–(23) and (27) will be used in numeric calculations.

3 Combined generation of magnetic and chromomagnetic fields

To calculate the strengths of the combined generated magnetic and chromomagnetic fields we use the perturbative computation method in [7]. First of all we find the strengths of the fields x and y when the quark and the s-quark contributions (v'_q) are divided in two parts, $v'_q(x,\beta) = v'_q\,|_{y\to 0}$ and $v'_q(y,\beta) = v'_q\,|_{x\to 0}$, where $v'_q(x,\beta)$ is the quark and s-quark contribution in the case of a single magnetic field, and $v'_q(y,\beta)$ is the one in the presence of a chromomagnetic field only.

Now let us rewrite v' in (15) as follows:

$$v'(\bar{x},\bar{y}) = v_1(\bar{x}) + v_2(\bar{y}) + v_3(\bar{x},\bar{y}), \tag{29}$$

where $\bar{x} = x + \delta x$, $\bar{y} = y + \delta y$, and δx and δy are the field corrections connected with the interfusion effect of the fields in the quark and s-quark sector.

Since the mixing of fields due to quark and s-quark loop is weak (this is justified in numeric calculations) one can assume that $\delta x \ll 1$ and $\delta y \ll 1$, and one can write

$$v_1(\bar{x}) = v_1(x+\delta x) = v_1(x) + \frac{\partial v_1(x)}{\partial x}\delta x, \tag{30}$$

$$v_2(\bar{y}) = v_2(y+\delta y) = v_2(y) + \frac{\partial v_2(y)}{\partial y}\delta y, \tag{31}$$

$$v_3(\bar{x},\bar{y}) = v_3(x+\delta x, y+\delta y) = v_3(x,y). \tag{32}$$

After simple transformations we can find δx and δy:

$$\delta x = \frac{\dfrac{\partial v_3(x,0)}{\partial x} - \dfrac{\partial v_3(x,y)}{\partial x}}{\dfrac{\partial^2 v_1(x)}{\partial x^2}},$$

$$\delta y = \frac{\dfrac{\partial v_3(0,y)}{\partial y} - \dfrac{\partial v_3(x,y)}{\partial y}}{\dfrac{\partial^2 v_2(y)}{\partial y^2}}. \tag{33}$$

Hence we obtain $\bar{x} = x + \delta x$ and $\bar{y} = y + \delta y$.

These results on the field strengths determined by means of numeric investigation of the total EP are summarized in Tables 1 and 2.

In Tables 1 and 2, in the first column we show the inverse temperature. In the second one the strengths of magnetic and chromomagnetic fields are adduced for the case of the quark and the s-quark EP describing each of the fields separately. The next column gives the field corrections in the case of the total quark and s-quark EP. The fourth column presents the relative value of the corrections. The following column gives the resulting strengths of magnetic ($\bar{x} = x+\delta x$) and chromomagnetic ($\bar{y} = y+\delta y$) fields, respectively. In the last column the strengths of the generated fields in the SM are given for comparison [7].

As is seen, the increase of the inverse temperature leads to decreasing strengths of the generated fields. This dependence is well in accordance with the picture of the universe's cooling.

Table 1. The strengths of generated magnetic field

	MSSM				SM
β	x	δx	$\delta x/x$, %	$\bar{x}$	$\bar{x}$
0.1	0.3813	1.58×10^{-2}	4.14	0.3971	0.7000
0.2	0.10021	2.45×10^{-3}	2.44	0.10265	0.20075
0.3	0.046199	7.19×10^{-4}	1.56	0.046917	0.069945
0.4	0.026804	1.97×10^{-4}	0.73	0.027000	0.039964
0.5	0.017675	1.19×10^{-4}	0.67	0.017794	0.029953
0.6	0.0120559	5.20×10^{-5}	0.43	0.0121079	0.0199508
0.7	0.0086022	2.82×10^{-5}	0.33	0.0086303	0.0099620
0.8	0.0065687	1.72×10^{-5}	0.26	0.0065859	0.0099381
0.9	0.0052535	1.13×10^{-5}	0.22	0.0052648	0.0099759
1.0	0.0043400	8.10×10^{-6}	0.19	0.0043481	0.0099643

Table 2. The strengths of the generated chromomagnetic field

	MSSM				SM
β	y	δy	$\delta y/y$, %	$\bar{y}$	$\bar{y}$
0.1	0.510146	5.28×10^{-6}	0.0010	0.510151	0.800301
0.2	0.133035	1.73×10^{-6}	0.0013	0.133037	0.199761
0.3	0.0603172	8.85×10^{-7}	0.0015	0.0603181	0.0899012
0.4	0.0347127	5.20×10^{-7}	0.0015	0.0347132	0.0499116
0.5	0.0225367	3.59×10^{-7}	0.0016	0.0225371	0.0398880
0.6	0.0161563	2.26×10^{-7}	0.0014	0.0161565	0.0299018
0.7	0.0115808	1.53×10^{-7}	0.0013	0.0115810	0.0199558
0.8	0.00859328	1.19×10^{-7}	0.0014	0.00859340	0.0199267
0.9	0.00672412	9.94×10^{-8}	0.0015	0.00672422	0.0098830
1.0	0.00547797	9.01×10^{-8}	0.0016	0.00547806	0.0098250

From the above analysis it follows that in the considered temperature interval the presence in the system of both fields leads to increasing of each of them in contrast with the SM case. In the latter one the strengths of the combined fields are decreased as compared to the separate generation. This is the consequence of the s-quark loop contributions depending as the quark loops on both of the fields.

With temperature decreasing this effect becomes less pronounced and disappears at comparably low temperatures, $\beta \sim 1$.

4 Discussion

Let us discuss the results obtained. As we elaborated within the EP including the one-loop and the daisy diagrams, in the MSSM at high temperatures both the magnetic and chromomagnetic fields have to be generated. This vacuum is stable, as follows from the absence of imaginary terms in the EP minima.

If quark and s-quark loops are discarded, both of the fields can be generated separately. All these states are stable, due to the magnetic mass $\sim g^2(gH)^{1/2}T$ of the transversal gauge field modes. As was already shown in [7], the imaginary part arises for field strengths larger than the ones generated in the SM. As we have seen, the strengths

Table 3. The contribution of quarks, s-quarks, s-leptons and charginos to the EP

β	quarks	s-quarks	s-leptons	charginos
0.1	8.09496×10^{-2}	1.29860×10^{-2}	1.08818×10^{-2}	1.90295×10^{-3}
0.2	5.51654×10^{-3}	5.13130×10^{-4}	6.28152×10^{-4}	1.16324×10^{-4}
0.3	1.13926×10^{-3}	6.77157×10^{-5}	1.13423×10^{-4}	2.20561×10^{-5}
0.4	3.78190×10^{-4}	1.51991×10^{-5}	3.28547×10^{-5}	6.66057×10^{-6}
0.5	1.60125×10^{-4}	4.51358×10^{-6}	1.24231×10^{-5}	2.60925×10^{-6}
0.6	8.11555×10^{-5}	1.64583×10^{-6}	5.07024×10^{-6}	1.09729×10^{-6}
0.7	4.16467×10^{-5}	6.18990×10^{-7}	2.28091×10^{-6}	5.06211×10^{-7}
0.8	2.31108×10^{-5}	2.55271×10^{-7}	1.18256×10^{-6}	2.68001×10^{-7}
0.9	1.42387×10^{-5}	1.18144×10^{-7}	6.76190×10^{-7}	1.55895×10^{-7}
1.0	9.48895×10^{-6}	5.96475×10^{-8}	4.14475×10^{-7}	9.68813×10^{-8}

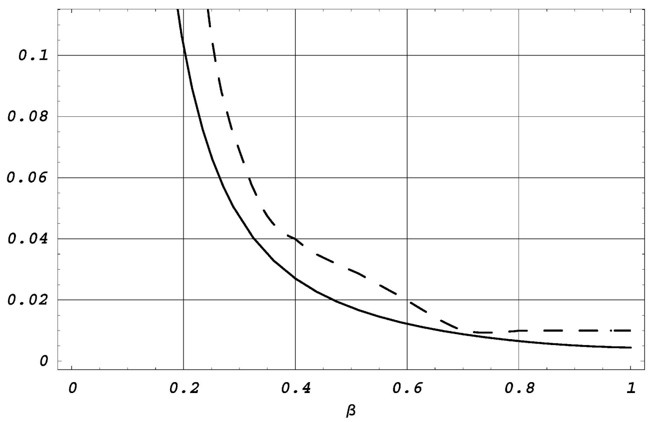

Fig. 1. The dependences of the strengths of the generated magnetic fields (**H**) on the inverse temperature (β). The solid line is the magnetic field strength in the MSSM and the dashed line in the SM

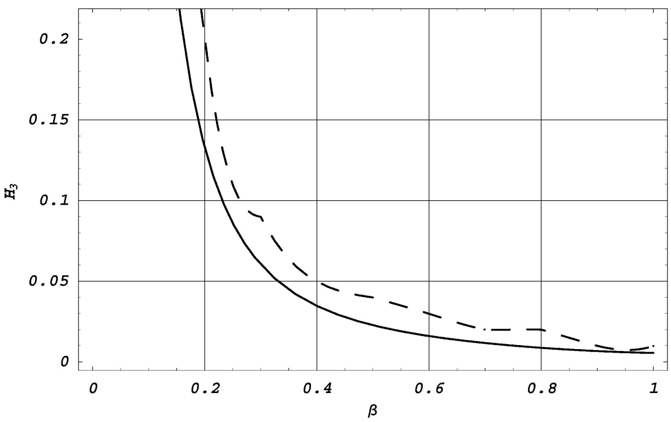

Fig. 2. The dependences of the strengths of the generated chromomagnetic field ($\mathbf{H}_3$) on the inverse temperature (β). The solid line is the chromomagnetic field strength in the MSSM and the dashed line is that of the SM

of the generated fields are reduced due to the s-particle sector of the MSSM. This vacuum state is more stable as compared to the SM case. In Table 3 the contribution of quark and s-particle sectors of MSSM to the EP is shown.

The result on the stabilization of the charged gauge field spectra is very important. It has relevance not only to the problem of the consistent description of the generation of magnetic fields but also to the related problem on the symmetry behavior in external magnetic fields investigated in the MSSM recently in [9,13].

As is seen from Figs. 1 and 2, presenting the results of numeric computations within the exact EP, the strengths of the generated fields are increasing when the temperature is increasing. It is also found that the dynamics of curves obtained in the SM [7] are in good agreement with our numeric calculations.

For the ground state possessing the magnetic and the chromomagnetic fields it is reasonable to expect the existence of these fields in the electroweak transition epoch for both the SM and the MSSM. The state is stable in the whole considered temperature interval. The imaginary part in the EP exists for the external fields being much stronger than the strengths of the spontaneously generated ones. The mixing of magnetic and chromomagnetic fields arising from the quark and the s-quark sectors of the EP is weak. In the MSSM, the change of the field minima in the inclusion of the field mixing does not exceed 4 per cents. In the SM these values do not exceed 2 per cents. This is due to the strong dependence of the s-quark loop on the strengths of both fields.

During the universe's cooling the strengths of the generated fields are decreasing. This is in agreement with what is expected in cosmology.

One of the consequences of the results obtained is the presence of a strong chromomagnetic field in the early universe, in particular, at the electroweak phase transition and, probably, until the deconfinement phase transition. The influence of this field on the phase transitions may bring about new insights into these phenomena. As our estimate showed, the chromomagnetic field is as strong as the magnetic one. So the role of strong interactions in the early universe in the presence of the field needs more detailed investigations as compared to what is usually assumed [3].

We would like to notice that in the literature devoted to investigations of the quark–gluon plasma in the deconfinement phase carried out by non-perturbative methods the vacuum magnetization at high temperature has not been accounted for (see, for instance, the recent paper of [25] and references therein). From the point of view of the present analysis (as well as other studies carried out already in perturbation theory [4–6]) these investigations are incomplete. The generation of the chromomagnetic field at high temperature has to be taken into consideration.

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